Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐ Candidate Number ☐☐☐☐

Friday 14 June 2019

Afternoon | Paper Reference **9MA0-32**

Mathematics

Advanced

Paper 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*
- Unless otherwise stated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P63359A

1/1/1/1/C2/1/

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})\,\mathrm{m}$.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

(a) To get acceleration from velocity, we differentiate :

$$v = 6ti - 5t^{\frac{3}{2}}j$$

M1A1 $$a = \frac{dv}{dt} = 6i - \frac{15}{2}t^{\frac{1}{2}}j$$

★ simple differentiation: $\frac{dy}{dx} = nx^{n-1}$

Now substitute $t = 4$

$$\left.\frac{dv}{dt}\right|_{t=4} = 6i - \frac{15}{2}(4)^{\frac{1}{2}}j = (6i - 15j)\,ms^{-2} = a$$ A1

(b) To get displacement from velocity, we integrate:

$$v = 6ti - 5t^{\frac{3}{2}}j$$

M1 $$d = \int v\,dt = \int 6ti - 5t^{\frac{3}{2}}j\,dt$$

★ Simple Integration: $\int x^n\,dx = \frac{1}{n}x^{n+1} + c$

$$= \frac{6}{2}t^2 i - \frac{5}{\frac{5}{2}}t^{\frac{5}{2}}j$$

$$= 3t^2 i - 2t^{\frac{5}{2}}j + c$$ A1

To get $\underline{\underline{c}}$ use given info: $t=0$, $d = (-20i + 20j)\,m$

Substitute:

$$(-20i + 20j) = 3(0)^2 i - 2(0)^{\frac{5}{2}}j + c$$

$$\therefore \quad c = (-20i + 20j)\,m$$

Hence: $$d = (3t^2 - 20)i + (20 - 2t^{\frac{5}{2}})j$$

Now substitute $t=4$ to get the position vector.

$$d = (3(4)^2 - 20)i + (20 - 2(4)^{\frac{5}{2}})j$$

$$d = (28i - 44j)\,m$$ A1

Question 1 continued

(Total for Question 1 is 6 marks)

2. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

(a) Use suvat

$\cancel{s}$

$u = (-i + 4j)\,ms^{-1}$

$v = v$

$a = (2i - 3j)\,ms^{-2}$

$t = T$

Use formula $v = u + at$

$v = (-i + 4j) + T(2i - 3j)$ M1A1

$v = (2T - 1)i + (4 - 3T)j$

↳ this moves in direction $(3i - 4j)$.

$\therefore$ Find $\frac{i}{j}$ ratios and equate them

$\frac{3}{-4} = \frac{2T-1}{4-3T}$ M1

↓ solve

$3(4 - 3T) = -4(2T - 1)$

$12 - 9T = -8T + 4$

$T = 8$ A1

(b) Use suvat again (motion A → B)

$s = \cancel{s}$

$u = (-i + 4j)\,ms^{-1}$

$\cancel{v}$

$a = (2i - 3j)\,ms^{-2}$

$t = 4$

Use formula $s = ut + \frac{1}{2}at^2$ M1

$s = (-i + 4j)(4) + \frac{1}{2}(2i - 3j)(4)^2$ A1

$= (-4i + 16j) + 8(2i - 3j)$

$= (16 - 4)i + (16 - 24)j$

$= 12i - 8j$

↳ this is displacement

To get distance, use Pythagoras' theorem:

$|s| = \sqrt{12^2 + (-8)^2} = \sqrt{208}$ M1

$= 4\sqrt{13} = 14.4m$ to 3sf A1

Question 2 continued

(Total for Question 2 is 8 marks)

3.

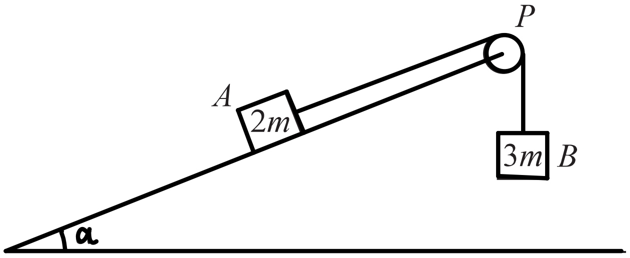


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$ μ

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

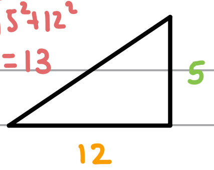
(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

$\tan\alpha = \frac{5}{12}$

$\sqrt{5^2+12^2}$
$= 13$

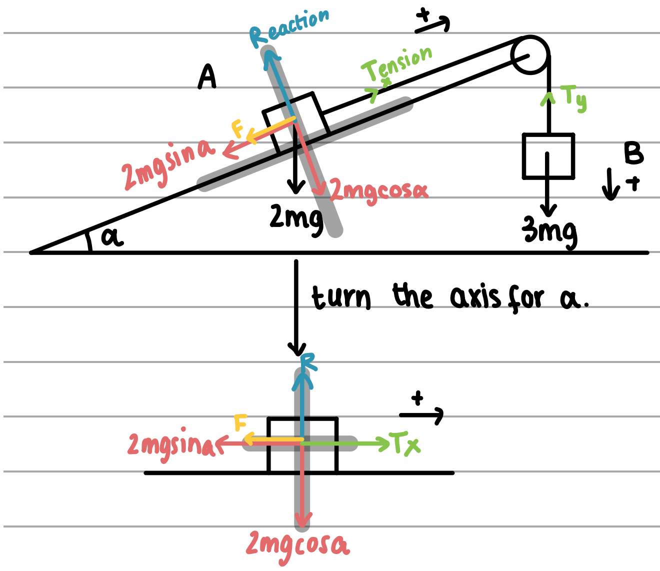


$\therefore \cos\alpha = \frac{12}{13}$

$\sin\alpha = \frac{5}{13}$

Question 3 continued

(a) Let's draw the forces:



→ We know $T_x = T_y$ as string is light and acceleration is the same as the string is inextensible

From Eq. 2 : $a = \frac{3mg - T}{3m}$

substitute in Eq 1. ↓

Solve to make T → the subject $T - \frac{26}{13}mg = 2\cancel{m}\left(\frac{3mg - T}{3\cancel{m}}\right)$ M1

$$T = 2mg - \frac{2}{3}T + \frac{\overset{2}{\cancel{26}}}{\cancel{13}}mg$$

$$\frac{5}{3}T = 4mg$$

$$T = \frac{12mg}{5}$$ hence shown A1

For A:

$\Sigma F_y = 0$ (only moves along surface)

$R = 2mg\cos\alpha = 2mg \times \frac{12}{13} = \frac{24}{13}mg$ B1

$\Sigma F_x = ma$ (it's accelerating up the plane)

$T_x - F - 2mg\sin\alpha = 2mga$ M1

$F \to F = \mu R$ since it's moving. So:

$F = \frac{2}{\cancel{3}_1} \times \frac{\overset{8}{\cancel{24}}}{13}mg = \frac{16}{13}mg$ B1

$T_x - \frac{16}{13}mg - 2mg\left(\frac{5}{13}\right) = 2ma$ A1

↓

Eq. 1 $T_x - \frac{26}{13}mg = 2ma$

For B:

$\Sigma F_y = ma$ (as it moves upwards)

Eq. 2 $3mg - T_y = 3ma$ M1 A1

(b) For A to come to rest, the friction (upwards) has to be larger than the horizontal component of the weight:

$F = \frac{16}{13}mg$ $W_x = \frac{10}{13}mg$ (2mgsinα)

$\frac{16}{13}mg > \frac{10}{13}mg$ M1

$\therefore$ $F > W_x$ $\therefore$ stays at rest. A1

(c) Extensible string

String has weight (not light) B1

Pulley is rough

Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4.

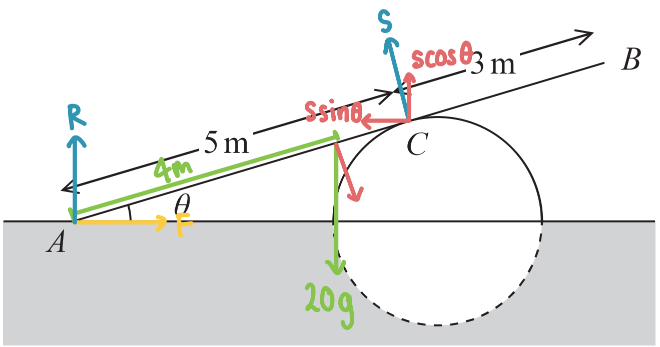


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

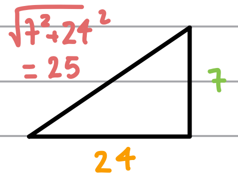
The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

$\tan\alpha = \frac{7}{24}$



$\therefore \cos\alpha = \frac{24}{25}$

$\sin\alpha = \frac{7}{25}$

(a) Because the drum is smooth B1

Question 4 continued

☆ See diagram for drawn out forces

(b) ∴ we need friction and reaction force.

Let's first find the magnitude of S (reaction force at C) using moments

$\overset{\circlearrowright +}{\Sigma M_A} = 0$ as it's in equilibrium

$4 \times 20g\cos\theta = 5 \times S$

$\therefore \quad S = \frac{4}{\cancel{5}} \times \overset{4}{\cancel{20}}g \times \frac{24}{25} = \frac{16\times24}{25}g = \frac{384}{25}g$ value of S

M1A1

Now let's solve vertically + horizontally ($\Sigma F = 0$ as it's in equilibrium)

vertically

$R + S\cos\theta = 20g$

$R = 20g - \frac{384}{25}g \times \frac{24}{25} = \frac{3284}{625}g$ value of R

M1A1

horizontally

$F = S\sin\theta \qquad F = \frac{384}{25}g \times \frac{7}{25} = \frac{2688}{625}g$ value of F

M1A1

} Now that we got these two we can use Pythagoras' theorem to get the resultant at A. M1

Pythagoras' theorem to get |Resultant|:

$$\sqrt{\left(\frac{3284}{625}g\right)^2 + \left(\frac{2688}{625}g\right)^2} \quad \text{M1}$$

$= 66.5$ N to 3sf. A1

(c) The magnitude of the normal reaction at C would decrease. This is because the clockwise moment about A decreases if center of mass is closer to A. B1

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

5.

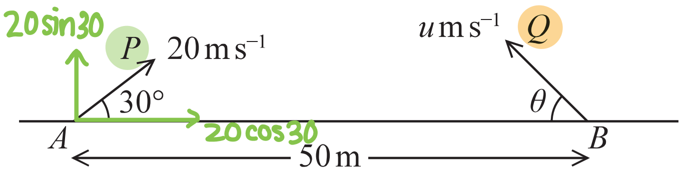


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

(a) Motion of P.

Horizontally

$u = 20\cos 30 = \overset{10}{\cancel{20}}\frac{\sqrt{3}}{\cancel{2}} = 10\sqrt{3}$ B1

(+↑) Vertically use suvat

$u = 20\sin 30 = 20 \times \frac{1}{2} = 10$

$v = v$

$a = -g$

$t = 2$ M1

use formula $v = u + at$

$v = 10 - 2g$

$v = -9.6$ A1

to get speed use Pythagoras' theorem:

$\sqrt{(\ 0\sqrt{3})^2 + (-9.6)^2} = 19.8 ms^{-1}$ M1

to get the angle of velocity to the horizontal:

$\gamma = \tan^{-1}\left(\frac{9.6}{10\sqrt{3}}\right)$ M1

$\gamma = 29°$

∴ velocity is $19.8 ms^{-1}$ at 29° to the horizontal A1

Question 5 continued

(b) Sum of horizontal distances = 50m M1

P at t=2:
$s = ut$
$s = 2\times10\sqrt{3} = 20\sqrt{3}$

Q at t=2
$s = ut$
$s = 2u\cos\theta$

$\therefore 20\sqrt{3} + 2u\cos\theta = 50$

Eq1. $u\cos\theta = 25-10\sqrt{3}$ A1

The vertical distances at t=2 must be equal since the two collide. M1

(↑+) P use suvat	Q use suvat	Use Formula $s=ut+\frac{1}{2}at^2$
$s = s$	$s = s$	$s = 2\times(20\sin30)+\frac{1}{2}(-g)(2)^2$
$u = 20\sin30$	$u = u\sin\theta$	$s = 2\times(u\sin\theta)+\frac{1}{2}(-g)(2)^2$ } equate
$\cancel{v}$	$\cancel{v}$	
$a = -g$	$a = -g$	$\cancel{2}(20\times\frac{1}{2}) - \cancel{2g} = \cancel{2}u\sin\theta - \cancel{2g}$
$t = 2$	$t = 2$	Eq.2 $u\sin\theta = 10$ A1

Now we use Eq1 and Eq2 to get θ:

divide:
$$\frac{\cancel{u}\sin\theta = 10}{\cancel{u}\cos\theta = 25-10\sqrt{3}} \longrightarrow \tan\theta = \frac{10}{25-10\sqrt{3}}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\theta = \tan^{-1}\left(\frac{10}{25-10\sqrt{3}}\right) = 52.478$

$\theta = 52.5$ to 3sf

Substitute θ back into Eq.2 to get u: M1

$u\sin(52.4775\ldots) = 10$

$u = \frac{10}{\sin(52.4775\ldots)} = 12.6085\ldots$

$\theta = 52.5$ to 3sf

$u = 12.6$ to 3sf A1

(c) The model ignores the size of the balls

The model ignores wind effects B1

The model ignores spin on the ball

The model ignores the fact that the balls are not particles

Question 5 continued

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 13 marks)

TOTAL FOR MECHANICS IS 50 MARKS

BLANK PAGE